

Heavy and light quarks in the instanton vacuum

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Abstract. Assuming the gluon field is well approximated by instanton configurations we derive a light quarks determinant and calculate its contribution to the specific heavy quarks correlators – namely, the heavy quark propagator and heavy quark-antiquark correlator, receiving the instanton generated light-heavy quarks interaction terms contributions. With these knowledge we calculate the light quark contribution to the interaction between heavy quarks, which might be essential for the properties of a few heavy quarks systems.

1 Introduction.

The physics of the heavy mesons and baryons with open and hidden heavy quarks is very reach and hot topic. Understanding the heavy-meson physics is important for evaluation of the components of the CKM -matrix, verification of the Standard Model and probing the physics beyond it, as well as production of different exotic meson states. Currently the experiments with B - and D -mesons are intensively studied by Belle [1], BaBar [2] and CDF collaborations, where unprecedented integrated luminosities were achieved, as well as neutrino-production of open and hidden charm in neutrino-hadron processes studied by K2K [3], MiniBoone [4], NuTeV [5] and Minerva [6] collaborations.

Theoretically, in pre-QCD era some success was achieved by the quantum-mechanical models which use effective potentials to describe heavy hadrons and their excitations (see e.g. [7] and references therein). However, such description inevitably introduces undefined phenomenological constants. The relation of these constants to QCD parameters is quite obscure: due to interaction with gluons and virtual light quark pairs all the constants contain nonperturbative dynamics. The numerical values of these constants are determined from fits to experimental data, which limits the predictive power of such models.

An advanced version of the potential model is NRQCD [8], however in this model light quarks and their interactions with heavy quarks via gluons is done in a phenomenological way. For this reason it is limited to description of systems with two heavy quarks. Alternatively, the heavy mesons are described in the Heavy Quark Effective Theory (HQET) proposed in [9], which treats the heavy mesons using the pQCD methods but does not take into account nonperturbative effects.

We propose to study the heavy quark physics in the framework of the instanton vacuum model. This model was developed in [10] and provided a consistent description of the light mesons physics [14].

One of the most prominent advances of the instanton vacuum model is the correct description of the spontaneous breaking of the chiral symmetry ($S\chi SB$), which is responsible for properties of most hadrons and nuclei [15]. The $S\chi SB$ is due to specific properties of QCD vacuum, which is known to be one of the most complicated objects due to perturbative as well as non-perturbative fluctuations and is a very important object of investigations by methods of Nonperturbative Quantum Chromo Dynamics (NQCD). In the instanton picture $S\chi SB$ is due to the delocalization of single-instanton quark zero modes in the instanton medium. One of the advantages of the instanton vacuum is that it is characterized by only two parameters: the average instanton size $\rho \sim 0.3$ fm and the average inter-instanton distance $R \sim 1$ fm. These essential numbers were suggested in [16] and were derived from $\Lambda_{\overline{MS}}$ in [10]. These values were recently confirmed by lattice measurements [17].

In case of the heavy quarks, the instanton vacuum description was discussed in [12,13]. For the heavy quarks even the charmed quark mass $m_c \sim 1.5$ GeV is larger than the typical parameters of the instanton media—the inverse instanton size $\rho^{-1} \approx 600$ MeV and the interinstanton distance $R^{-1} \approx 200$ MeV and thus the quark mass determines the dynamics of the heavy quarks.

2 Light quark determinant with the quark sources term.

Instanton vacuum field is assumed as a superposition of N_+ instantons and N_- antiinstantons:

$$A_\mu(x) = \sum_I^{N_+} A_\mu^I(\xi_I, x) + \sum_A^{N_-} A_\mu^A(\xi_A, x). \quad (1)$$

Here $\xi = (\rho, z, U)$ are (anti)instanton collective coordinates—size, position and color orientation (see reviews [10,18]). The main parameters of the model are the average inter-instanton distance R and the average instanton size ρ . The

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estimates of these quantities are

$$\begin{aligned}\rho &\simeq 0.33 \text{ fm}, R \simeq 1 \text{ fm}, (\text{phenomenological}) [10,18], \\ \rho &\simeq 0.35 \text{ fm}, R \simeq 0.95 \text{ fm}, (\text{variational}) [10], \\ \rho &\simeq 0.36 \text{ fm}, R \simeq 0.89 \text{ fm}, (\text{lattice}) [17]\end{aligned}\quad (2)$$

and have $\sim 10 - 15\%$ uncertainty.

Our main approximation is the interpolation formula for the light quark propagator in a single instanton field:

$$\begin{aligned}S_i &= S_0 + S_0 \hat{p} \frac{|\Phi_{0i} \rangle \langle \Phi_{0i}|}{c_i} \hat{p} S_0, \\ S_0 &= \frac{1}{\hat{p} + im}, \quad c_i = im \langle \Phi_{0i} | \hat{p} S_0 | \Phi_{0i} \rangle.\end{aligned}\quad (3)$$

The advantage of this interpolation is shown by the projection of S_i to the zero-modes:

$$S_i | \Phi_{0i} \rangle = \frac{1}{im} | \Phi_{0i} \rangle, \quad \langle \Phi_{0i} | S_i = \langle \Phi_{0i} | \frac{1}{im} \quad (4)$$

as it must be, while the similar projection of S_i given by [10] has a wrong component, negligible only in the $m \rightarrow 0$ limit.

Summation of the re-scattering series leads to the light quark propagator in the instanton vacuum:

$$S = S_0 - S_0 \sum_{i,j} \hat{p} | \Phi_{0i} \rangle \langle \Phi_{0i} | \frac{1}{B} | \Phi_{0j} \rangle \langle \Phi_{0j} | \hat{p} S_0, \quad (5)$$

where $B = \hat{p} S_0 \hat{p}$. Here $\tilde{\text{Tr}}$ means the trace on the flavor and only on zero-mode ($| \Phi_{0j} \rangle$) space. The explicit form of the matrix $B(m)$ on the flavor and only on zero-modes ($| \Phi_{0j} \rangle$) space is:

$$B_{ij}^{fg} = \delta_{fg} \langle \Phi_{0i} | \hat{p} S_0 \hat{p} | \Phi_{0j} \rangle. \quad (6)$$

Then the low-frequency part of the light quark determinant [14] is

$$\text{Det}_{\text{low}}[m] = \det B(m). \quad (7)$$

Making few further steps [14] we get the fermionized representation of low-frequencies light quark determinant in the presence of the quark sources, which is relevant for our problems, in the form:

$$\begin{aligned}\text{Det}_{\text{low}} \exp(-\xi^+ S \xi) &= \int \prod_f D\psi_f D\psi_f^\dagger \prod_{\pm,f}^{N_\pm} V_{\pm,f}[\psi^\dagger, \psi] \\ &\times \exp \int_f \sum (\psi_f^\dagger (\hat{p} + im_f) \psi_f + \psi_f^\dagger \xi_f + \xi_f^\dagger \psi_f),\end{aligned}\quad (8)$$

where

$$\begin{aligned}V_{\pm,f}[\psi^\dagger, \psi] &= i \int d^4x (\psi_f^\dagger(x) \hat{p} \Phi_{\pm,0}(x; \xi_\pm)) \\ &\times \int d^4y (\Phi_{\pm,0}^\dagger(y; \xi_\pm) (\hat{p} \psi_f(y))).\end{aligned}\quad (9)$$

The light quark partition function $Z[\xi_f, \xi_f^+]$ is given by the averaging of $\text{Det}_{\text{low}} \exp(-\xi^+ S \xi)$ over the collective coordinates of the instantons ξ_\pm as:

$$Z[\xi_f, \xi_f^+] = \int D\xi \text{Det}_{\text{low}} \exp(-\xi^+ S \xi), \quad D\xi = \prod_\pm d\xi_\pm.$$

The averaging over collective coordinates ξ_\pm is a rather simple procedure, since factorized form of the Eq. (8) and the low density of the instantons ($\pi^2 (\frac{\rho}{R})^4 \sim 0.1$). These one allows us to average over positions and orientations of the instantons independently.

Light quark partition function at $N_f = 1$ and $N_\pm = N/2$ is exactly given by

$$\begin{aligned}Z[\xi, \xi^+] &= \exp \left[-\xi^+ (\hat{p} + i(m + M(p)))^{-1} \xi \right] \\ &\times \exp \left[\text{Tr} \ln \frac{\hat{p} + im + iM(p)}{\hat{p} + im} + N \ln \frac{N/2}{\lambda} - N \right] \\ N &= \text{tr} \frac{iM(p)}{\hat{p} + i(m + M(p))}, \quad M(p) = \frac{\lambda}{N_c} (2\pi\rho F(p))^2\end{aligned}\quad (10)$$

Here the form-factor $F(p)$ is given by Fourier-transform of the zero-mode. The coupling λ and the dynamical quark mass $M(p)$ are defined by the Eq. (11).

At $N_f = 2$, $N_\pm = N/2$ and saddle-point approximation (no meson loops contribution)

$$\begin{aligned}Z[\xi_f, \xi_f^+] &= \exp \left[- \sum_f \xi_f^+ (\hat{p} + im_f + iM_f(p))^{-1} \xi_f \right] \\ &\times \exp \left[N \ln \frac{N/2}{\lambda} - N - \frac{V\sigma^2}{2} + \sum_f \text{Tr} \ln \frac{\hat{p} + im_f + iM_f(p)}{\hat{p} + im_f} \right].\end{aligned}\quad (12)$$

Here λ, σ and dynamical quark mass

$$M(p) = \frac{\lambda^{0.5}}{2g} (2\pi\rho)^2 F^2(p) \sigma, \quad g^2 = \frac{(N_c^2 - 1) 2N_c}{2N_c - 1}$$

are defined from the Eqs.

$$N = \frac{1}{2} \text{Tr} \frac{iM_f(p)}{\hat{p} + im_f + iM_f(p)} = \frac{1}{2} \sigma^2. \quad (13)$$

In general, at $N_f > 2$, and in the saddle-point approximation (no meson loops contribution) $Z[\xi_f, \xi_f^+]$ has a similar form as the Eqs. (10, 12).

3 Light quark propagator

The propagator is defined as

$$S = \int DA \text{Det}(\hat{P} + im) \frac{1}{\hat{P} + im}, \quad \hat{P} = \hat{p} + \hat{A}. \quad (14)$$

In the instanton vacuum model $A \approx \sum_i A_i$, where A_i are instantons and $DA \approx D\xi$. Then, accordingly Eq. (11) the light quark propagator is:

$$S = \frac{1}{\hat{p} + i(m + M(p))}. \quad (15)$$

Pobylitsa [11] neglected by the quark determinant:

$$S_{Pob} = \int D\zeta \frac{1}{\hat{P} + im} \quad (16)$$

and derived the Eq.:

$$S_{Pob}^{-1} = S_0^{-1} + \int D\zeta \sum_i (S_{Pob} - \hat{A}_i^{-1})^{-1}, \quad (17)$$

where it was applied large N_c argumentation. Representing $S_{Pob}^{-1} - S_0^{-1} = \Sigma$, it was found the Eq.:

$$\Sigma = \frac{N}{2VN_c} \text{tr}_c \sum_{\pm} \int dz_{\pm} \frac{\hat{p}|\Phi_{0,\pm}\rangle\langle\Phi_{0,\pm}| \hat{p}}{\Sigma_0} + O\left[\left(\frac{N}{VN_c}\right)^2\right],$$

$$\Sigma_0 = \langle\Phi_{0,\pm}|\Sigma|\Phi_{0,\pm}\rangle \quad (18)$$

and finally ($m = 0$ case) the solution for the dynamical quark mass:

$$M_{Pob}^2(k) = \frac{N}{4VN_c} \frac{(2\pi\rho)^4 F^4(k)}{\int \frac{d^4q}{(2\pi)^4} \frac{(2\pi\rho)^4 F^4(q)}{q^2}} \quad (19)$$

corresponding to the Eq.

$$4N_c \int \frac{d^4q}{(2\pi)^4} \frac{M_{Pob}^2(q)}{q^2} = \frac{N}{V} \quad (20)$$

The Eq. (11) for the λ ($m = 0$ case) has explicit form

$$4N_c \int \frac{d^4q}{(2\pi)^4} \frac{M^2(q)}{q^2 + M^2(q)} = \frac{N}{V}. \quad (21)$$

As we see, the difference between Eqs. (20) and (21) is only in the denominators. This one is due to the account of the quark determinant in the derivation of the Eq. (21) (and Eq. (11)). In the following we will use the Eq. (11) for the dynamical quark mass $M(k)$.

Any N_f case in the saddle-point approximation has no essential difference with the present case $N_f = 1$.

4 Heavy quark propagator.

At the ref. [12] it was considered the Eq. for the heavy quark propagator in the line similar the Eq. (17). Our aim here is to extend the approach [12] taking in-to account the light quarks contribution at $N_f = 1$ case. So, define the heavy quark propagator as:

$$S_H = \frac{1}{Z} \int D\psi D\psi^\dagger \prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^\dagger, \psi] e^{\int \psi^\dagger (\hat{p} + im) \psi} w[\psi, \psi^\dagger],$$

$$w[\psi, \psi^\dagger] = \int \frac{D\zeta}{\prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^\dagger, \psi]} \prod_{\pm}^{N_{\pm}} V_{\pm}[\psi^\dagger, \psi] \frac{1}{\theta^{-1} - \sum_i a_i},$$

$$w_{\pm} = \frac{1}{\theta^{-1} - a_{\pm}}, \quad \langle t|\theta|t' \rangle = \theta(t - t'), \quad (22)$$

$$\langle t|\theta^{-1}|t' \rangle = -\frac{d}{dt} \delta(t - t'), \quad a_i(t) = iA_{i\mu}(x(t)) \frac{d}{dt} x_{\mu}(t).$$

In the $w[\psi, \psi^\dagger]$ the measure of the integration has a factorized form $\prod_{\pm}^{N_{\pm}} \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]}$ as in the Eq. (17). It provide the way for the extension of this Eq.. Extended Eq. with the account of the light quarks has a form:

$$w^{-1}[\psi, \psi^\dagger] = \theta^{-1} + \int \prod_{\pm}^{N_{\pm}} \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]} \sum_i (w[\psi, \psi^\dagger] - \hat{A}_i^{-1})^{-1}. \quad (23)$$

Again, we have the approximate solution of this Eq. as:

$$w^{-1}[\psi, \psi^\dagger] - \theta^{-1} = \frac{N}{2} \sum_{\pm} \int \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]} V_{\pm}[\psi^\dagger, \psi] (\theta - a_{\pm}^{-1})^{-1} + O(N^2/V^2)$$

$$= -\frac{N}{2} \sum_{\pm} \int \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]} V_{\pm}[\psi^\dagger, \psi] \frac{1}{\theta} (w_{\pm} - \theta) \frac{1}{\theta} + O(N^2/V^2)$$

$$\equiv -\frac{N}{2} \sum_{\pm} \frac{1}{\bar{V}_{\pm}[\psi^\dagger, \psi]} \Delta_{H,\pm}[\psi^\dagger, \psi] + O(N^2/V^2)$$

and finally we get

$$S_H = \left[\frac{1}{\theta^{-1} - \lambda \sum_{\pm} \Delta_{H,\pm}[\frac{\delta}{\delta\xi}, \frac{\delta}{\delta\xi^\dagger}]} e^{-\xi^+ (\hat{p} + i(m+M(p)))^{-1} \xi} \right]_{|\xi=\xi^+=0} \quad (25)$$

If to neglect by overlapping quark loops, then

$$S_H^{-1} \approx \left[\left(\theta^{-1} - \lambda \sum_{\pm} \Delta_{H,\pm}[\frac{\delta}{\delta\xi}, \frac{\delta}{\delta\xi^\dagger}] \right) e^{\xi^+ (\hat{p} + i(m+M(p)))^{-1} \xi} \right]_{|\xi=\xi^+=0}$$

$$= \theta^{-1} - \frac{N}{2VN_c} \sum_{\pm} \int d^4z_{\pm} \text{tr}_c (\theta^{-1} (w_{\pm} - \theta) \theta^{-1}). \quad (26)$$

The Eq. (26) exactly coincide with the similar one from [12].

Now re-write the Eq. (25) introducing heavy quark fields Q, Q^\dagger :

$$S_H = e^{[-\text{tr} \ln(\hat{p} + i(m+M(p)))]} \int D\psi D\psi^\dagger DQ DQ^\dagger Q Q^\dagger \quad (27)$$

$$\times \exp \left[\psi^\dagger (\hat{p} + i(m+M(p))) \psi + Q^\dagger \left(\theta^{-1} - \lambda \sum_{\pm} \Delta_{H,\pm}[\psi^\dagger, \psi] \right) Q \right]$$

$$\times \exp \left[-\text{tr} \ln \left(\theta^{-1} - \lambda \sum_{\pm} \Delta_{H,\pm}[\psi^\dagger, \psi] \right) \right],$$

where last exponent represent the (negligible) contribution of the heavy quark loops, while the second one has the heavy and light quarks interaction action, explicitly represented by

$$-\lambda \sum_{\pm} Q^\dagger \Delta_{H,\pm}[\psi^\dagger, \psi] Q =$$

$$= -i\lambda \sum_{\pm} \int d^4z_{\pm} \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} e^{i(k_2 - k_1)z_{\pm}} (2\pi\rho)^2 F(k_1) F(k_2)$$

$$\times \left[\frac{1}{N_c^2} \psi^+(k_1) \frac{1 \pm \gamma_5}{2} \psi(k_2) Q^\dagger \text{tr}_c (\theta^{-1} (w_{\pm} - \theta) \theta^{-1}) Q \right.$$

$$+ \frac{1}{32(N_c^2 - 1)} \psi^+(k_1) (\gamma_\mu \gamma_\nu \frac{1 \pm \gamma_5}{2}) \lambda^i \psi(k_2) \text{tr}(\tau_\mu^\mp \tau_\nu^\pm \lambda^j)$$

$$\times Q^\dagger \text{tr}_c (\theta^{-1} (w_{\pm} - \theta) \theta^{-1} \lambda^j) \lambda^i Q \left. \right]. \quad (28)$$

We see that the heavy-light quarks interactions terms has a form of the product of the colorless currents of a heavy and light quarks together with similar term of the colorful currents product. The structure of these currents are defined by the instanton color orientation integration, while the instanton position integration provide energy-momentum conservation in the interaction vertex.

At the $N_f > 1$ case we have an interaction vertex with N_f pairs of a light quark legs and the pair of a heavy quark legs. The specific structure of the interaction is defined again by instanton color orientation and will be much more reach then at $N_f = 1$ case. We expect that the action generated by the instantons will have reach symmetry properties related to light and heavy quarks sectors both. Namely, it appear the light-heavy quarks interaction terms leading to the specific traces of the light quarks chiral symmetry in light-heavy quarks systems.

5 Heavy quark anti-quark system.

Now it is considered the correlator for this system again with the account of ($N_f = 1$ case) light quark contribution:

$$\begin{aligned} < T|C(L_1, L_2)|0 > = \frac{1}{Z} \int D\psi D\psi^\dagger \left\{ \prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^\dagger, \psi] \right\} \quad (29) \\ & \times \exp \int (\psi^\dagger (\hat{p} + im)\psi) < T|W[\psi, \psi^\dagger]|0 >, \\ < T|W[\psi, \psi^\dagger]|0 > = \int \frac{D\zeta}{\left\{ \prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^\dagger, \psi] \right\}} \left\{ \prod_{\pm}^{N_{\pm}} V_{\pm}[\psi^\dagger, \psi] \right\} \\ \text{Tr} < T| \left(\theta^{-1} - \sum_i a_i^{(1)} \right)^{-1} |0 > < 0| \left(\theta^{-1} - \sum_i a_i^{(2)} \right)^{-1} |T >. \end{aligned}$$

Here the correlator is a Wilson loop along the rectangular contour $L \times r$, where the sides L_1, L_2 are parallel to x_4 axes and separated by the distance r . The $a^{(1)}, a^{(2)}$ are the projections of the instantons onto the lines L_1, L_2 . In the ref. [12] this correlator was considered within the approach similar to the Eq. (17) of the ref. [11] but without a light quarks.

The argumentation, which provided the derivation of the Eq (23), is applicable to the present case and leads to the similar Eq..

$$\begin{aligned} W^{-1}[\psi, \psi^\dagger] &= \\ &= w_1^{-1}[\psi, \psi^\dagger] \otimes w_2^{-1,T}[\psi, \psi^\dagger] - \frac{N}{2} \sum_{\pm} \int \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]} \\ &\times V_{\pm}[\psi^\dagger, \psi] \left(w_1[\psi, \psi^\dagger] - a_{\pm}^{(1)-1} \right)^{-1} \otimes \left(w_2[\psi, \psi^\dagger] - a_{\pm}^{(2)-1} \right)^{-1,T}, \end{aligned} \quad (30)$$

where, superscript T means the transposition, \otimes – tensor product. This Eq. has an approximate solution:

$$\begin{aligned} W^{-1}[\psi, \psi^\dagger] &= w_1^{-1}[\psi, \psi^\dagger] \otimes w_2^{-1,T}[\psi, \psi^\dagger] \quad (31) \\ &- \frac{N}{2} \sum_{\pm} \int \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]} V_{\pm}[\psi^\dagger, \psi] \\ &\times \left(\theta^{-1} \left(w_{\pm}^{(1)} - \theta \right) \theta^{-1} \right) \otimes \left(\theta^{-1} \left(w_{\pm}^{(2)} - \theta \right) \theta^{-1} \right)^T + O(N^2/V^2). \end{aligned}$$

and

$$\begin{aligned} w_1^{-1}[\psi, \psi^\dagger] &= \theta^{-1} - \\ &- \frac{N}{2} \sum_{\pm} \frac{d\zeta_{\pm}}{\bar{V}_{\pm}[\psi^\dagger, \psi]} V_{\pm}[\psi^\dagger, \psi] \theta^{-1} (w_{\pm}^{(1)} - \theta) \theta^{-1} + O(N^2/V^2) \\ &= \theta^{-1} - \frac{N}{2} \sum_{\pm} \frac{1}{\bar{V}_{\pm}[\psi^\dagger, \psi]} \mathcal{A}_{H,\pm}^{(1)}[\psi^\dagger, \psi] + O(N^2/V^2) \end{aligned} \quad (32)$$

and similar for the $w_2^{-1}[\psi, \psi^\dagger]$.

From previous calculations we see that the lowest orders on $\frac{N}{N_c V}$ in $C(L_1, L_2)$ are given by the integration over ψ, ψ^\dagger of the $W^{-1}[\psi, \psi^\dagger]$. Here it was neglected by overlapping quark loops. Then, we have the new interaction term between heavy quarks located on the lines L_1 and L_2 due to exchange of the light quarks between them. Explicitly the integration of the first term in $W^{-1}[\psi, \psi^\dagger]$ over ψ, ψ^\dagger leads to:

$$\begin{aligned} &\frac{1}{Z} \int D\psi D\psi^\dagger \left\{ \prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^\dagger, \psi] \right\} \exp \int \psi^\dagger (\hat{p} + im)\psi \quad (33) \\ &\times w_1^{-1}[\psi, \psi^\dagger] \otimes w_2^{-1,T}[\psi, \psi^\dagger] = \left(\theta^{-1} - \lambda \sum_{\pm} \mathcal{A}_{H,\pm}^{(1)} \left[\frac{\delta}{\delta \xi}, \frac{\delta}{\delta \xi^+} \right] \right) \\ &\otimes \left(\theta^{-1} - \lambda \sum_{\pm} \mathcal{A}_{H,\pm}^{(2)} \left[\frac{\delta}{\delta \xi}, \frac{\delta}{\delta \xi^+} \right] \right)^T e^{-\xi^+ (\hat{p} + i(m+M(p)))^{-1} \xi} \Big|_{\xi=\xi^+=0}. \end{aligned}$$

Light quarks generated potential is given by

$$\begin{aligned} V_{lq} &= \left(\lambda \sum_{\pm} \mathcal{A}_{H,\pm}^{(1)} \left[\frac{\delta}{\delta \xi_1}, \frac{\delta}{\delta \xi_1^+} \right] \right) \otimes \left(\lambda \sum_{\pm} \mathcal{A}_{H,\pm}^{(2)} \left[\frac{\delta}{\delta \xi_2}, \frac{\delta}{\delta \xi_2^+} \right] \right)^T \\ &\times e^{-\xi_2^+ (\hat{p} + i(m+M(p)))^{-1} \xi_1 - \xi_1^+ (\hat{p} + i(m+M(p)))^{-1} \xi_2} \Big|_{\xi=\xi^+=0}. \end{aligned} \quad (34)$$

The range of this potential is controlled by dynamical light quark mass $M \sim 350$ MeV and might be important for the heavy quarkonium states properties.

6 Conclusion.

Approximating the gluon field by the instanton configurations it was derived the low-frequency part of the light quark determinant in the presence of quark sources. It was provided the calculation of the instanton generated light-heavy quarks interaction terms and the heavy quark propagator with the account of the light quark determinant together with the QCD instanton vacuum properties at the $N_f = 1$ case. With these knowledge it was calculated the light quark contribution to the interaction between heavy quarks. The extension of this approach to $N_f > 1$ case is obvious and provide the possibility for the detailed investigation of the role of the light quarks chiral symmetry and its spontaneous breaking for the heavy and heavy-light quarks systems. The estimations of the light quark contributions to their properties are on the way.

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